

INTRODUCTION TO CONCEPT OF PORE-SHAPE FACTOR FOR POROUS SPHERES IN THE STUDY OF EXTRACTION RATES

RASHMI R. KESHVANI

Department of Mathematics, Sarvajani College of Engineering & Technology, Surat, Gujarat, India

ABSTRACT

In a liquid a satisfactory method of defining the diffusivity so that it will be independent of relative composition, has yet to be devised. The recent development of effective methods for the calculation of mass –transfer rates from particle surfaces, now brings forward the need of considering pore diameters, permeabilities and porosities. Knowledge of liquid diffusion rates in porous solids has applications in solid-liquid extraction and in catalysis. Piret E. L., Ebel R.A. Kiang C.T. Armstrong W. P. aimed to investigate about liquid diffusion rates which would be of value in the study of actual systems involving porous carrier solids and to obtain idealized extraction systems. In their work, they presented theoretical expressions and experimental data for extraction from single capillaries, from beds of uniform glass beads and from inert porous spheres carrying single-phase solutions of soluble substances. They introduced the concept of pore shape factor. Pore shape factor is equal to the ratio of the actual to the theoretical extraction time or for an actual sphere of measured radius R , an equivalent, idealized sphere of radius KR , ($K > 1.0$) can be postulated, whose structure offers no resistance to diffusion. The square of this constant, K^2 , will be called the pore shape factor. This factor is used in interpreting and correlating the extraction data. Here in this paper the author intends to interpret the results, obtained in their investigations and to derive a simple formula to determine this pore shape factor. For this purpose, Euler's Summation Formula and Abel's identity are applied to the analytic solutions obtained for their experiments. This approximation is obtained only for small values of $\frac{Dt}{R^2}$, where D is diffusivity, t is time and R is measured radius of sphere. The formula is verified for experimental data. It shows nice agreement with the experimental data.

KEYWORDS: Extraction, Pore Shape Factor, Euler's Summation Formula, Abel's Identity and Big oh Notation

INTRODUCTION

Related to investigations of extraction rates, a series of experiments was carried out on the batch extraction of single phase solutes from porous spheres by Edgar L. Piret, R. A. Ebel, C.T. Kiang and W.P. Armstrong (University of Minnesota, Minneapolis, Minnesota).[1] Aqueous solutions of potassium chloride, sodium chloride, potassium chromate, copper sulfate, and acetic acid were extracted from three types of carrier solids of increasing complexity: single capillaries, tubes containing beds of small glass-beads and porous alumina spheres. The transfer mechanism was verified as being diffusional. Equations were derived for the extraction of a single solute from single capillaries and from spheres in the batch-wise operation. Validity of the equation of batch-wise operation was verified by extractions from porous alumina spheres under several conditions of concentration, solvent volume and temperature.

The influence of an inert porous solid upon diffusion rate has not been extensively investigated. Cady and Williams [2] studied diffusion of urea through wood fibers. They stated that the diffusion rate was decreased principally by a mechanical blocking effect of the cell structure; the effect of cell wall friction on molecular motion was of much less importance.

Boucher, Brier and Osburn [3] found that the extraction of soybean oil from porous chaplets was a purely diffusional operation whose rate was decreased by the nature of the solid. They suggested the factor affecting rate to be the mechanical blocking by the solid, the drag on molecule due to the proximity to cell walls and the tortuous path through the solid.

Piret E. L., Ebel R.A. Kiang C.T. Armstrong W. P. studied several facts, which affect liquid diffusion rates and which would be of value in the study of actual systems involving porous carrier solids. In their experiments, they studied extraction from single capillaries, from beds of uniform glass beads and from inert porous spheres carrying single-phase solutions of soluble substances. The new thing is that, they introduced the concept of pore shape factor. Pore shape factor is equal to the ratio of the actual to the theoretical extraction time or for an actual sphere of measured radius R , an equivalent, idealized sphere of radius KR , ($K > 1.0$) can be postulated, whose structure offers no resistance to diffusion. The square of this constant, K^2 , will be called the pore shape factor. This factor is used in interpreting and correlating the extraction data.

RESEARCH OBJECTIVES

It was the purpose of the work of the team of Piret E. L., Ebel R.A. Kiang C.T. and Armstrong W. P. to obtain on idealized extraction systems further basic information on liquid diffusion rates. They conducted experiments using: (1) Single Capillary Investigations and (2) Porous Sphere Investigations.

In this paper the data obtained during their investigations are interpreted mathematically and a simple formula to predict pore shape factor is derived.

SINGLE CAPILLARY INVESTIGATION

Procedure and Data

A straight capillary tube of constant bore and known internal length and volume, but with one end sealed off, was filled with a binary solution of known composition. The capillary was held in a vertical position with the open end up, and a very slow stream of pure solvent was allowed to flow at right angles to the longitudinal axis of the capillary. The temperature was maintained at 25.0 ± 0.1 °C. After an elapsed time t , the capillary was removed and the solution inside the capillary was washed into a flask or crucible and analyzed volumetrically or gravimetrically. Amount and composition of the solution now inside the capillary were determined. Generally several such experiments were performed and the data presented as E (per cent of extractable material remaining unextracted) vs. time of extraction t .

They used the capillary–tube method to determine the integral diffusivity for a number of systems. Their data for $KCl - H_2O$ solutions ($C_0 = 4$ N) are given below:

Table 1

No.	$t/L^2, sec/cm^2$	(100 - E)%
1	51.87×10^3	95.1
2	36.50	85.8
3	28.97	80.4
4	17.03	63.2
5	8.805	45.9
6	3.689	28.8
7	2.838	24.9
8	1.242	15.9
9	1.160	15.8
10	0.741	12.2

Interpretations

Assuming that the solvent concentration is constant, and C_A is the solute concentration and D is diffusivity of a binary liquid system, the governing partial differential equation will be,

$$\frac{\partial C_A}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial C_A}{\partial x} \right) \quad (1)$$

with the boundary conditions

$$C_A = C_0 \text{ at } t = 0, 0 = x = L$$

$$C_A = 0 \text{ at } x = L, t > 0$$

$$D \frac{\partial C_A}{\partial x} = \frac{\partial}{\partial t} \int_0^L C_A dx ; \text{ at } x = L, t > 0$$

Assuming, diffusivity D , to be constant, the solution [1],[4],[5] to equation (1), will be

$$E = \frac{800}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp \left(\frac{-(2n-1)^2 \pi^2 D t}{4L^2} \right) \quad (2)$$

For sufficiently large values of t , equation(2) may be approximated as

$$E = \frac{800}{\pi^2} \left(\exp \left(\frac{-\pi^2 D t}{4L^2} \right) + \frac{1}{9} \exp \left(\frac{-9\pi^2 D t}{4L^2} \right) \right) \quad (3)$$

(Considering only first two terms, as contribution from the terms after that will be negligible.)

$$\text{Taking, } y = \exp \left(\frac{-\pi^2 D t}{4L^2} \right), \text{ equation (3) results in, } E = \frac{800}{\pi^2} \left(y + \frac{1}{9} y^9 \right)$$

Applying this equation for the data for which E is between 25% to 75%, That is for:

$$(100 - E) = 63.2, 45.9, 28.8, \text{ (for fourth, fifth and sixth observation in table),}$$

$$y = 0.4539, 0.6648, 0.8488, \text{ respectively.}$$

$$\text{Moreover, } y = \exp \left(\frac{-\pi^2 D t}{4L^2} \right) \Rightarrow D = \frac{4 \times \ln y}{\pi^2 \times t / L^2}.$$

So, diffusivity $D = 1.87977 \times 10^{-5}, 1.87922 \times 10^{-5}, 1.801 \times 10^{-5} \text{ cm}^2/\text{sec}$ respectively.

Based on the capillary experiments, they determined integral diffusivity of potassium chloride to be $1.87 \times 10^{-5} \text{ cm}^2/\text{sec}$. (Literature value of diffusivity of KCl is $1.84 \times 10^{-5} \text{ cm}^2/\text{sec}$.)

POROUS SPHERE INVESTIGATIONS

Procedure and Data

A series of experiments was carried out on the batch extraction of single phase solutes from porous aluminum spheres. This was done by filling capillaries of the porous spheres by salt solutions and determining the rate of extraction into a measured volume of solvent. The purpose of these experiments was to determine the effect of porous medium upon extractions. Single-phase solute is contained in porous sphere and the sphere is placed into a finite quantity of agitated solvent. As the diffusion proceeds, the concentration in the outside solution rises.

The following table shows their experimental data for extraction of aqueous Potassium Chloride from porous spheres:

Table 2

Salt KCl	Initial Outside Concentration = 0					
Temperature °c	25		25		25	
Initial Inside Concentration g/ml	0.224		0.114		0.307	
Solution Ratio a	25		25		25	
	t (Minutes)	(100-E) %	t (Minutes)	(100-E) %	t (Minutes)	(100-E) %
	5	16.3	10	24.1	20	33.8
	15	28.2	30	40.4	60	49.4
	45	46.9	60	54.0	150	71.6
	240	82.6	180	78.0	240	82.3
	300	85.4	375	90.8	360	89.5
	480	92.3	540	95.6	600	95.0
	1140	98.8	1380	99.0	720	95.4

Interpretation

The equation for symmetrical diffusion from a porous sphere of porosity P is

$$\frac{\partial Pc}{\partial t} = D \left(\frac{\partial^2 Pc}{\partial r^2} + \frac{2}{r} \frac{\partial Pc}{\partial r} \right)$$

That is

$$\frac{\partial c}{\partial t} = D \left(\frac{\partial^2 c}{\partial r^2} + \frac{2}{r} \frac{\partial c}{\partial r} \right) \quad (4)$$

Defining a new variable,

$u = cr$, (4) converts into

$$\frac{\partial u}{\partial t} = D \left(\frac{\partial^2 u}{\partial r^2} \right) \quad (5)$$

The boundary conditions are,

$$u = 0 \text{ at } r = 0$$

$$u = c_0 r \text{ when } t = 0$$

$$u = cr \text{ when } t > 0$$

$$c = c_\infty = C_\infty \text{ when } t = \infty$$

$$\left(\frac{\partial c}{\partial r} \right)_{r=R} = \left(\frac{\partial C}{\partial t} \right) \text{ V at } r = R$$

where $c = c(r, t)$ = Concentration of inside solution at any point r and time t , g./cc

C = Concentration of outside solution at any time, g./cc

R = Radius of the sphere

The solution [1],[5] to this problem is

$$E = \frac{200a^2}{3} \sum_{p=1}^{\infty} \frac{1}{\left[1+a \left(1+\frac{am_p^2}{9} \right) \right]} \exp \left(\frac{-m_p^2 Dt}{K^2 R^2} \right) \quad (6)$$

Here m_p 's, are positive roots of the equation $\tan x = \frac{x}{1 + \frac{a}{3}x^2}$

a = solution ratio, volume of outside solution per unit volume of solution in porous sphere.

K^2 = pore shape factor, to be determined yet.

In practice, porous spheres with all these characteristics were difficult to be obtained. Here an actual sphere of radius R which usually had tortuous, constricted capillaries is considered to be equivalent to an idealized sphere of radius KR . ($K > 1.0$) The increased length of path and the throttling effect of constrictions provide resistance to diffusion. In fluid flow a valve is considered to be equivalent in resistance to a certain length of straight pipe. Here an actual sphere of radius R which usually has tortuous, constricted capillaries is considered to be equivalent to an idealized sphere of radius KR . ($K > 1.0$) The square of this constant, K^2 will be called the pore shape factor. This factor is also equal to the ratio of the actual to theoretical time, based on an ideal sphere, for a given per cent extraction.

In their experiments, they made early runs with an initial inside concentration of zero and a $3N$ outside concentration for the purpose of determining pore shape factors of alumina spheres. Results were plotted on semi-log paper with E per cent unextracted as the ordinate and time t in minutes as the abscissa. They drew curves, taking $K^2 = 4, K^2 = 7, K^2 = 9, K^2 = 14$, and substituting an integral diffusivity of KCl to be $1.87 \times 10^{-5} \text{ sq.cm./sec}$, which was determined by capillary experiments, in equation (6). For large value of t , experimental results were close to curve for $K^2 = 9$, otherwise all points fell close to a single theoretical curve for

$K^2 = 7$. [1] Thus it may be seen that variations of inside concentration have no effect on the data. This is an agreement with equation (6). They accepted value of K^2 to be 7.

Results may be verified in other way also as follows: For large value of t , equation (6) can be approximated as,

$$E = \frac{200a^2}{3} \left(\frac{1}{1+a\left(1+\frac{am_1^2}{9}\right)} \exp\left(\frac{-m_1^2 Dt}{K^2 R^2}\right) \right) \quad (7)$$

considering only first term of infinite series, because then onwards contribution from the terms will be negligible.

For $a = 25$, first positive root of $\tan x = \frac{x}{1 + \frac{25}{3}x^2}$ is $m_1 = 3.179$ (It is close to)

Substituting $a = 25, D = 1.87 \times 10^{-5} \text{ cm}^2/\text{sec}, R = 0.5985 \text{ cm}$, (as given in [1])

and $m_1 = 3.179$ in (7), now expression will be

$$E = \frac{200 \times 625}{3} \left(\frac{1}{1+25\left(1+\frac{25 \times (3.179)^2}{9}\right)} \exp\left(\frac{-(3.179)^2 \times 1.87 \times 10^{-5} \times t}{K^2 \times (0.5985)^2}\right) \right)$$

$$\Rightarrow E = \frac{125000}{3} \left(\frac{1}{727.8084} \exp\left(\frac{-52.7587 \times 10^{-5} \times t}{K^2}\right) \right)$$

$$\Rightarrow K^2 = \left(\frac{52.7587 \times 10^{-5} \times t}{\ln\left(\frac{125000}{3 \times E \times 727.8084}\right)} \right) \quad (8)$$

Using (8), the obtained results, are as follows.

Table 3

t (in Minutes)	t (in Seconds)	E %	K²
240	240× 60	17.4	6.379
300	300× 60	14.6	6.950
375	375× 60	9.2	6.493
480	480×60	7.7	7.574
1140	1140×60	1.2	9.337

It can be noticed that the values of K^2 obtained here, are close to 7, for the first four values in the table and close to 9, for the last value, similar to the results, obtained using theoretical curves.

DERIVATIVE OF A SIMPLE FORMULA TO DETERMINE K^2

Precession and accuracy seem better for small values of t. Instead of using approximation for large value of t, approximation for small values of t, would be better. Here in this paper the author intends to derive a simpler formula to determine value of K^2 . This formula will be valid for only small values of $\frac{Dt}{R^2}$. It will be derived taking help of Euler’s Summation Formula and Abel’s Identity. Statements of these theorems [6] are as follows:

Euler’s Summation Formula

If f has a continuous derivative f' on the interval $[y, x]$, where $0 < y < x$, then

$$\sum_{y < n \leq x} f(n) = \int_y^x f(t)dt + \int_y^x (t - [t])f'(t)dt + f(x)([x] - x) - f(y)([y] - y) \tag{9}$$

Abel’s Identity

For any arithmetical function $a(n)$, let $A(x) = \sum_{n \leq x} a(n)$, where $A(x) = 0$, if $x < 1$.

Assume f has a continuous derivative on the interval $[y, x]$, where $0 < y < x$. Then

$$\sum_{y < n \leq x} a(n)f(n) = A(x)f(x) - A(y)f(y) - \int_y^x A(t)f'(t)dt. \tag{10}$$

Derivation

With the aim to apply these wonderful theorems to the solution (6), here it is required to determine an arithmetical function, which can be utilized as $a(n)$ and a function f , which has continuous derivative. For $a = 25$, values of first 10 roots of $\tan x = \frac{x}{1 + \frac{a}{9}x^2}$ are:

$$m_1 = 3.179, m_2 = 6.302, m_3 = 9.438, m_4 = 12.576, m_5 = 15.715,$$

$$m_6 = 18.856, m_7 = 21.997, m_8 = 25.137, m_9 = 28.275, m_{10} = 31.420.$$

It can be noticed that as n becomes large, $m_n \rightarrow n\pi$. Actually the difference between m_2 and 2π is also less than 0.018815 and gradually the difference goes on decreasing, remaining positive, as n increases.

So, $\frac{1}{\left[1+a\left(1+\frac{am_n^2}{9}\right)\right]}$ can be approximated by $\frac{1}{\left[1+a\left(1+\frac{a(n\pi)^2}{9}\right)\right]}$. That is, by $\frac{1}{\left[(1+a)+\frac{a^2}{9}n^2\pi^2\right]}$.

Also, as $\frac{1}{\left[1+a\left(1+\frac{am_n^2}{9}\right)\right]} \leq \frac{1}{\left[(1+a)+\frac{a^2}{9}n^2\pi^2\right]} < \frac{1}{\frac{a^2}{9}n^2\pi^2}$,

$$\sum \frac{1}{\left[1+a\left(1+\frac{amn^2}{9}\right)\right]} \leq \sum \frac{1}{\frac{a^2}{9}n^2\pi^2} = \frac{9}{a^2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{9}{a^2\pi^2} \times \frac{\pi^2}{6} = \frac{3}{2a^2}, \text{ confirms its convergence.}$$

So, here $\frac{1}{\frac{a^2}{9}n^2\pi^2}$ can serve as $a(n)$. As exponential functions have continuous derivatives,

$\exp\left(\frac{-n^2\pi^2Dt}{K^2R^2}\right)$ can serve here as $f(n)$.

Let y be a number between 0 and 1. So, $A(y) = \sum_{n < y} a(n) = 0$, which fulfills requirement demanded in the statement of the identity.

Let x be a very large number. So

$$\begin{aligned} A(x) &= \sum_{n \leq x} a(n) = \sum_{n=1}^{\infty} a(n) - \sum_{n > x} a(n) \\ &= \frac{3}{2a^2} - \frac{9}{a^2\pi^2} \sum_{n > x} \frac{1}{n^2} \end{aligned}$$

Now according to Euler's summation formula

$$\begin{aligned} \sum_{x < n < M} \frac{1}{n^2} &= \int_x^M \frac{dt}{t^2} - 2 \int_x^M \frac{t-[t]}{t^3} dt + \frac{[M]-M}{M^2} - \frac{[x]-x}{x^2}, \text{ where } M \text{ is very large number, tending to infinity.} \\ &= \left[\frac{-1}{t}\right]_x^M + O(x^{-2}) \\ &= -\left(\frac{1}{M} - \frac{1}{x}\right) + O(x^{-2}) \sim \frac{1}{x} \text{ as } M \text{ is tending to infinity.} \end{aligned}$$

$$\text{Here } -2 \int_x^M \frac{t-[t]}{t^3} dt + \frac{[M]-M}{M^2} - \frac{[x]-x}{x^2} = O(x^{-2}),$$

means that $\frac{-2 \int_x^M \frac{t-[t]}{t^3} dt + \frac{[M]-M}{M^2} - \frac{[x]-x}{x^2}}{x^{-2}}$ is bounded for all $x > 0$.

The symbol $O(x^{-2})$ represents an unspecified function of x which grows no faster than some constant times x^{-2} .

The symbol 'O' is known as big oh notation [6].

$$\text{Thus } A(x) = \frac{3}{2a^2} - \frac{9}{a^2\pi^2} \left(\frac{1}{x}\right) \quad (11)$$

Now using (11)

$$\begin{aligned} \sum_{y < n \leq x} \frac{9}{a^2n^2\pi^2} \exp\left(-\frac{n^2\pi^2Dt}{K^2R^2}\right) \\ = \left(\frac{3}{2a^2} - \frac{9}{a^2\pi^2x}\right) \times \exp\left(-\frac{x^2\pi^2Dt}{K^2R^2}\right) - A(y) \times \exp\left(-\frac{y^2\pi^2Dt}{K^2R^2}\right) - \int_y^x A(p)f'(p)dp \end{aligned} \quad (12)$$

As x is very large number, $\exp\left(-\frac{x^2\pi^2Dt}{K^2R^2}\right) \rightarrow 0$. As y is very small number $\exp\left(-\frac{y^2\pi^2Dt}{K^2R^2}\right) \rightarrow 1$.

$$\text{Here } A(p) = \frac{3}{2a^2} - \frac{9}{a^2\pi^2} \left(\frac{1}{p}\right) \text{ (using (11)) and } f'(p) = \exp\left(-\frac{p^2\pi^2Dt}{K^2R^2}\right) \times \frac{-2p\pi^2Dt}{K^2R^2}.$$

On substituting these results, now (12) yields,

$$\begin{aligned} \sum_{y < n \leq x} \frac{9}{a^2n^2\pi^2} \exp\left(-\frac{n^2\pi^2Dt}{K^2R^2}\right) &= - \int_y^x \left(\frac{3}{2a^2} - \frac{9}{a^2\pi^2} \left(\frac{1}{p}\right)\right) \exp\left(-\frac{p^2\pi^2Dt}{K^2R^2}\right) \times \frac{-2p\pi^2Dt}{K^2R^2} dp \\ &= - \int_y^x \left(\frac{3}{2a^2} \exp\left(-\frac{p^2\pi^2Dt}{K^2R^2}\right) \times \frac{-2p\pi^2Dt}{K^2R^2}\right) dp + \int_y^x \frac{9}{a^2\pi^2} \left(\exp\left(-\frac{p^2\pi^2Dt}{K^2R^2}\right) \times \frac{-2\pi^2Dt}{K^2R^2}\right) dp \end{aligned}$$

$$= -\frac{3}{2a^2} [\exp\left(-\frac{p^2\pi^2 Dt}{K^2 R^2}\right)]_y^x + \frac{9}{a^2} \int_y^x \exp\left(-\frac{p^2\pi^2 Dt}{K^2 R^2}\right) \left(\frac{-2Dt}{K^2 R^2}\right) dp \quad (13)$$

As x is very large and y is very small, there is no harm, if x is replaced by ∞ and y is replaced by 0^+ . Hence (13) yields

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{9}{a^2 n^2 \pi^2} \exp\left(-\frac{n^2 \pi^2 Dt}{K^2 R^2}\right) &= -\frac{3}{2a^2} [\exp\left(-\frac{p^2 \pi^2 Dt}{K^2 R^2}\right)]_{0^+}^{\infty} + \frac{9}{a^2 \pi^2} \int_{0^+}^{\infty} \exp\left(-\frac{p^2 \pi^2 Dt}{K^2 R^2}\right) \left(\frac{-2\pi^2 Dt}{K^2 R^2}\right) dp \\ &= -\frac{3}{2a^2} (0 - 1) + \frac{9}{a^2 \pi^2} \left(\int_{0^+}^{\infty} \exp(-s^2) \left(-\frac{2\pi\sqrt{Dt}}{KR}\right) ds \right) \\ &= \frac{3}{2a^2} - \frac{\sqrt{Dt}}{KR} \times \frac{9}{a^2 \pi} (2 \int_{0^+}^{\infty} \exp(-s^2) ds) \\ &= \frac{3}{2a^2} - \frac{\sqrt{Dt}}{KR} \times \frac{9}{a^2 \pi} \times \sqrt{\pi} \text{ as } (2 \int_{0^+}^{\infty} \exp(-s^2) ds) = \Gamma(1/2) = \sqrt{\pi} \\ &= \frac{3}{2a^2} - \frac{9}{a^2 K} \sqrt{\frac{Dt}{\pi R^2}} \end{aligned} \quad (14)$$

Using (14) in (6), result will be $E = \frac{200a^2}{3} \left(\frac{3}{2a^2} - \frac{9}{a^2 K} \sqrt{\frac{Dt}{\pi R^2}} \right) = 100 - \frac{600}{K} \sqrt{\frac{Dt}{\pi R^2}}$

That is, $E = 100 - \frac{600}{K} \sqrt{\frac{Dt}{\pi R^2}}$ (15)

As the minimum possible value of E is zero, the value of $\frac{600}{K} \sqrt{\frac{Dt}{\pi R^2}}$ can't exceed 100,

so this formula is valid for only small values of $\sqrt{\frac{Dt}{R^2}}$,

and $\frac{600}{K} \sqrt{\frac{Dt}{\pi R^2}} \leq 100 \Rightarrow \sqrt{\frac{Dt}{\pi R^2}} \leq \frac{K}{6} \Rightarrow \frac{Dt}{R^2} \leq \frac{\pi K^2}{36}$ (16)

Thus, for small values of $\sqrt{\frac{Dt}{R^2}}$, $K^2 = \frac{(600)^2 \times Dt / R^2}{(100-E)^2 \times \pi}$ (17)

Using (17) for small values of t ($t \leq 60$) from Table-2, results obtained are as follows.

Table 4

Initial Concentration Inside	t (in Seconds)	$\frac{Dt}{R^2}$	(100 - E)%	K^2
(1) 0.224	5×60	0.0156615	16.3	6.75479
	15×60	0.0469846	28.2	6.77033
	45×60	0.1409539	46.9	7.34318
(2) 0.114	10×60	0.0313231	24.1	6.17992
	30×60	0.0939693	40.4	6.59744
	60×60	0.1879385	54.0	7.38551
(3) 0.307	20×60	0.0626462	33.8	6.28368
	60×60	0.1879385	49.4	8.82499

CONCLUSIONS

- It can be noticed that except for last observation in Table-4, all the experimental data show good agreement. There might be any error in recording the last observation.
- Taking the mean of first seven observations, $K^2 = 6.75926$ with standard deviation $s = 0.46926$.
- It can be also noticed that initial inside concentration has no effect on extraction rate.
- It can be noticed that if $\frac{Dt}{R^2} \leq 0.09$, that is if $t < 30$ minutes, then there is better agreement in results.

Thus Formula (17) can be used to predict pore shape factor K^2 for small values of $\frac{Dt}{R^2}$.

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